Entanglement-breaking evolutions

Entanglement is a fundamental resource in quantum information that features in many well-known protocols (e.g., teleportation). Therefore, it is of interest to understand which evolutions of quantum systems preserve or destroy this resource. In this project, you will investigate so-called entanglement-breaking maps, i.e., maps that for any input state produce a separable output. You will focus on physical situations where the interactions between an open system under consideration and its environment are weak so that the dynamics can be well described by a Markovian evolution. In mathematical terms, this means that you will study (continuous) semigroups of quantum channels.

In this project, you will first learn about basic properties of the objects of interest (i.e., entanglement-breaking channels and semigroups of channels) by solving a collection of mathematical exercises. Once you are familiar with these notions and after some technical preparation, you will be guided through the proof of the following equivalence, which is the main goal of the project: An evolution described by a semigroup of quantum channels becomes entanglement-breaking after some finite time if and only if every initial state converges to a unique full-rank invariant state.

Your task will be to provide the details for the steps of the proof based on the instructions given. In doing so, you will encounter techniques from different areas relevant to quantum information, e.g., linear algebra and spectral theory. In particular, you will have an opportunity to apply what you have learned in the lecture on "Introduction to Quantum Information Processing". This will give you a better understanding of the structure of the set of separable states, of the behaviour of Markovian quantum evolutions, and it will bring you closer to the current research about these questions.

Reading material:

1. Leonid Gurvits and Howard Barnum. *Largest separable balls around the maximally mixed bipartite quantum state*. Phys. Rev. A, 66:062311, Dec 2002.

2. Alexander S Holevo. *Coding theorems for quantum channels*. arXiv preprint quant-ph/9809023, 1998.

3. L. Lami and V. Giovannetti. *Entanglement-saving channels*. Journal of Mathematical Physics, 57(3):032201, 2016.

4. Michael M Wolf. *Quantum channels & operations: Guided tour*. 2012. `http://www-m5.ma.tum.de/foswiki/pub/M5/Allgemeines/MichaelWolf/QChannelLecture.pdf`