Lieb-Robinson bounds and applications

How fast can information propagate through a quantum many-body system with local interactions? Lieb-Robinson bounds provide fundamental upper limits on the speed at which the dynamics of such a system can distribute initially localized information. In particular, these bounds give a "speed limit" called the *Lieb-Robinson velocity*.

Applications of Lieb-Robinson bounds are numerous; including multi-dimensional Lieb-Schultz-Mattis theorems, exponential clustering in gapped ground states, area laws, the analysis of quantum phases of matter, the complexity of states and many, many more.

In this project, you will learn how to formalize the question of information propagation in terms of basic quantum-mechanical concepts such as Heisenberg-evolved operators and operator norms of commutators. Following a list of instructions, you will then develop a mathematical proof of this fundamental result.

As applications you may consider consequences of Lieb-Robinson bounds to the problem of state preparation or to topological order.

In the first application you will show that certain highly entangled states such as the GHZ-state cannot be generated by a locally generated unitary in constant time from a product state. This result can also be reinterpreted a circuit depth lower bound on preparation circuits and thus immediately connects to complexity-theoretic questions in quantum computing.

Another application concerns Kitaev's toric code, a local stabilizer Hamiltonian on a 2D lattice of qubits whose ground states satisfy a condition called topological quantum order (TQO): no local observable can distinguish orthogonal ground states. While TQO is immediately connected to quantum error correction via the so-called Knill-Laflamme conditions, here you will study another key consequence of TQO derived from Lieb-Robinson bounds: preparing ground states of the toric code from a product state using locally generated unitary evolution requires a time scaling at least linearly in the (linear) system size.

Overall, this project will help familiarize yourself with a fundamental tool in quantum many-body systems and basic results in topological order.

Reading material:

1. Sergey Bravyi, Matthew B Hastings, and Frank Verstraete. *Lieb-Robinson bounds and the generation of correlations and topological quantum order.* Physical Review Letters, 97(5):050401, 2006.

2. A Yu Kitaev. *Fault-tolerant quantum computation by anyons.* Annals of Physics, 303(1):230, 2003.

3. Bruno Nachtergaele. *An introduction to quantum spin systems*, 2016. `https://www-m5.ma.tum.de/Allgemeines/MA5020_2016S` (Ch. 4-5).

4. Hal Tasaki. *Physics and Mathematics of Quantum Many-Body Systems.* Springer 2020 (Ch. 8.4).