Universal gate sets for quantum computation

In the standard (circuit) model of quantum computation, a typical computation proceeds by application of an n-qubit unitary U to a certain initial state and subsequent measurement of each qubit in the computational basis. The hardware of a universal quantum computer should therefore provide the capability of applying an arbitrary unitary U. In practice, however, control of quantum systems is severely limited by the specifics of the system under consideration and/or experimental limitations. For example, control pulses (realizing unitary evolutions) may only be applied along a certain subset of directions with a restricted range of frequencies and/or durations. Is such hardware sufficient to reach the desired goal of performing universal quantum computation? If so, how many operations are required to realize a given unitary U?

In this project, you will be given a (ficticious) device that implements a single-qubit unitary specified by a discrete set of control parameter values. Your task is to use (several copies of) this device to (approximately) realize an arbitrary n-qubit unitary U. In other words, you will generate an instruction set (a quantum circuit) for executing the corresponding quantum computation using the given hardware.

A key notion of interest here is that of a *universal gate set*, a finite family $\{V_j\}_{j=1}^N$ of (typically single- and two-qubit) unitaries (called *gates*) such that any unitary U can be approximately written as a product of a certain number L of gates, that is,

$$U \approx V_{i_1} \dots V_{i_L} \qquad \text{for some} \qquad (i_1, \dots, i_L) \in \{1, \dots, N\}^L \ . \tag{1}$$

Given such a universal gate set, we may ask about the relationship between the length L of the gate sequence (ultimately corresponding to the runtime of your computation) and the quality of approximation in (1). In addition, we want to efficiently (by classical computation) find a suitable sequence $(i_1, \dots, i_L)$. The so-called Solovay-Kitaev theorem provides answers to this problem and is the main topic of this project. The project will involve a collection of mathematical exercises. You will also gain experience programming in python by developing and implementing a compilation procedure as described above.

Reading material:

1. Teofilo F. Gonzalez. *Clustering to minimize the maximum intercluster distance.* Theoretical Computer Science, 38:293–306, 1985.

2. A. Harrow. Quantum Compiling. PhD thesis.

3. M. Nielsen and I. Chuang. *Quantum Computation and Quantum Information.* Cambridge University Press, 2000.

4. Maris Ozols. *The Solovay-Kitaev theorem.* `http://home.lu.lv/~sd20008/papers/essays/Solovay-Kitaev.pdf`, 2009.