Semidefinite programming in quantum information theory

The need to optimize arises frequently in everyday live. What is the fastest route from A to B? What is the cheapest hotel in Munich that provides breakfast and is at most 5 kilometers away from Marienplatz? When engineering quantum devices, the broad question that arises over and over again is how to best perform a certain information processing task given the experimental constraints at hand. Convex optimization techniques (and especially semidefinite programs (SDPs)) have proven to be a vital tool to compute the answer to these questions efficiently. In this project, you will be introduced to SDPs by putting on the hat of a quantum engineer who tries to design devices that peform the following three tasks.

1. Suppose your friend challenges you with the following game: He will prepare a quantum system described by one of the (mixed) states $\rho_1, \rho_2, \ldots, \rho_N$. You have to guess which state it is. If you guess correctly, he will pay for the next barbeque and otherwise you have to pay. What is the best way to measure the system to maximize your chances of guessing right?

2. You have another friend who studies abroad. When she left, she gave you a part of her most beautiful quantum system (which we know to be in state ρ_{AB}) to stay connected. The next time you talk to her, she tells you that studying abroad has changed her mind about the beauty of quantum states completely and that she now likes the pure state ϕ the most. Unfortunately, she did not bring any quantum equipment with her so she asks you to perform a quantum operation on your part of the system such that the overall state is as close as possible to her beloved state ϕ. How would you choose that quantum operation to make her as happy as possible?

3. A few childhood friends of yours like to play with Boolean functions. As an introduction to their game they want you to guess which function they are currently playing with. You choose an input, and ask them for some information on the output. For each piece of information they give you, you have to pay some amount of money, and since you do not want to end up being poor, you need to guess the function as soon as possible. How could you do that?

In this project, you will learn how to formulate these problems mathematically as an SDP and how to solve such an SDP with Matlab using the cvx package.

Reading material:

1. Stephen Boyd and Lieven Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.

2. L. Vandenbeghe and S. Boyd. *Semidefinite programming*. SIAM Review, 38:49–95, link, 1996.

3. J. Watrous. *Semidefinite programs for completely bounded norms*. Theory of Computing, 5(11):217–238, 2009.

4. J. Watrous. *Simpler semidefinite programs for completely bounded norms*. Chicago Journal of Theoretical Computing Science, 8:1–19, link, 2013.